All questions are 1 point, with 1 free point.

1. Barley-Fungus studies

(a) e.u.=chamber, o.u.=trays

Source	DF	denominator 1	MS	for	F	test
fungus	1	MS_chamber				
barley	2	MS_chamber				
fungus*barley	2	MS_chamber				
chamber	12					
tray	36					

(b) block=repetition, e.u.=chamber, o.u.=trays

Source	DF	denominator MS for F test
repetition	2	
fungus	1	MS_chamber
barley	2	MS_chamber
fungus*barley	2	MS_chamber
chamber	10	
trays	36	

Note: chamber could also be written repetition*barley*fungus, since the combination of repetition, barkey and fungus uniquely identifies each chamber and time of use.

(c) block=repetition, whole plot e.u.=chamber, split plot e.u.=trays

```
denominator MS for F test
Source
                    DF
repetition
                    2
                    1
fungus
                         MS_chamber
                    2
chamber
barley
                    2
                         MS_tray
                    2
fungus*barley
                         MS_tray
                    8
trays
```

Note: chamber could also be written repetition*fungus

(d) whole plot e.u.=chamber, split plot e.u.=trays

```
Source
                    DF
                         denominator MS for F test
                         no F test
fungus
                    1
chamber
                    0
                    2
barley
                         MS_tray
fungus*barley
                    2
                         MS_tray
trays
                   12
```

(e) No F test for fungus can be provided in design(d)

2. Pigs:

(a)
$$Var(\bar{y}_{i.}) = \sigma_{\alpha}^2 + \frac{\sigma_e^2}{n_i}$$
 MSLitter= $\frac{n}{I}\sigma_{\alpha}^2 + \sigma_e^2$ so $\sigma_{\alpha}^2 = 0.10175$ and $\sigma_e^2 = 0.35761$

(b)
$$\sigma_{\alpha}^2 = 0.10464$$
 and $\sigma_e^2 = 0.35698$

(c) they are not the same because this is not a balanced design

(d) correlation=
$$\frac{\sigma_{\alpha}^2}{\sigma_e^2 + \sigma_{\alpha}^2} = 0.227$$

- (e) both are 2.676
- (f) prediction for birth weight born to female 1 $\hat{y}_{1.} = \frac{\sigma_{\alpha}^2}{\sigma_e^2/10 + \sigma_{\alpha}^2} \bar{y}_{1.} + \frac{\sigma_e^2/10}{\sigma_e^2/10 + \sigma_{\alpha}^2} \bar{y}_{..} = 2.798$ This is the same as the value of coef() reported by lmer.
- (g) prediction for the mean of 20 birth weights born to female 1 $\hat{y}_{1.} = \frac{\sigma_{\alpha}^2}{\sigma_{e}^2/20 + \sigma_{\alpha}^2} \bar{y}_{1.} + \frac{\sigma_{e}^2}{\sigma_{e}^2/20 + \sigma_{\alpha}^2} \bar{y}_{..} = 2.816$

Note: I didn't ask this question as carefully as I might have. I intended to ask about predicting the mean for litter if it were calculated from 20 piglets. This is the same value no matter whether you are predicting the mean of 1 new piglet, the mean of 10 new piglets, or the mean of 20 new piglets. What matters is how precisely estimated is the sample average for litter 1 and the variability among litters.

(h)
$$P(Y_{1j} < 2.5) = P(\mu + \alpha_1 + \epsilon_{1j} < 2.5) = P(\epsilon_{1j} < 2.5 - \mu - \alpha_1) = 0.31$$
, where $\epsilon_{1j} \sim N(0, \sigma_e^2)$

(i)
$$P(Y_{ij} < 2.5) = P(\mu + \alpha_i + \epsilon_{ij} < 2.5) = P(\epsilon_{ij} + \alpha_i < 2.5 - \mu) = 0.40$$
, where $\epsilon_{ij} + \alpha_i \sim N(0, \sigma_e^2 + \sigma_o^2)$

3. Fungus

- (a) estimates are 1.16 and -1096 with s.e. 0.1283 and 377.6
- (b) 95% confidence interval for the mean slope is (-1838,-353)
- (c) -2.125
- (d) 0.294
- (e) same as (c)