Power in pictures

Consider t-test of difference of 2 means, μ_1, μ_2

$H_0: \mu_1 = \mu_2$ or $\mu_1 - \mu_2 = 0$

$H_A: \mu_1 \neq \mu_2$ or $\mu_1 - \mu_2 \neq 0$

under usual t-test assumptions
(independent observations, equal variances, normal errors)

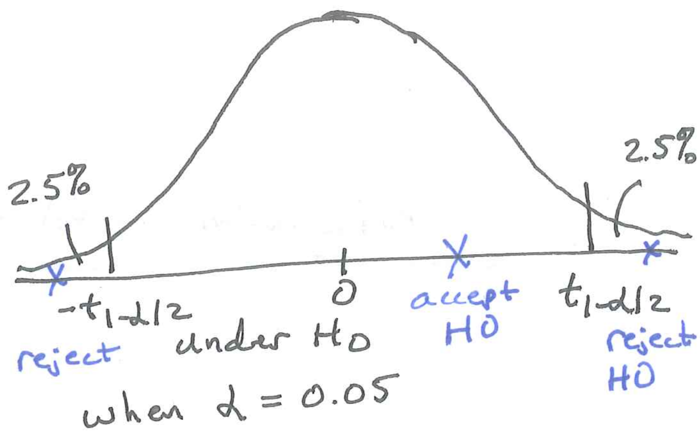
Data $\Rightarrow$ $\bar{Y}_1$, $\bar{Y}_2$, S

Calculate $T_{obs} = \dfrac{\bar{Y}_1 - \bar{Y}_2}{S\sqrt{2/n}}$

Reject H_0 (2 sided test)

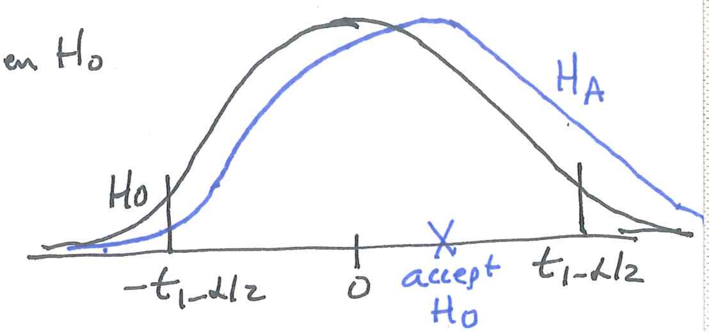
when $T_{obs} \geq t_{1-\alpha/2,\,df}$

or $T_{obs} \leq -t_{1-\alpha/2,\,df}$



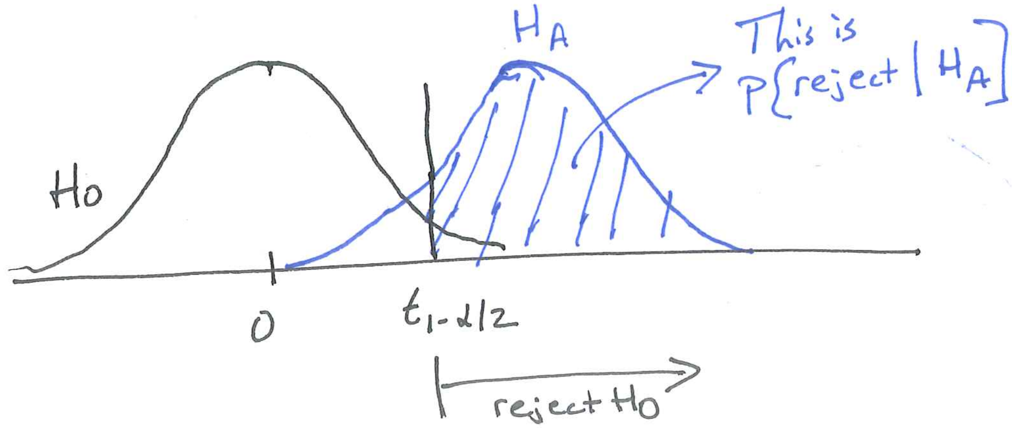
[Stat theory $\Rightarrow$ Tobs has very nice properties:
distribution doesn't depend on μ_1, μ_2, σ
probabilites can be computed]

what happens when H_A is true ($\mu_1 - \mu_2 \neq 0$)
small difference example
have a distribution of Tobs given Ho
and another given H_A
X result is an error!



want to find $P[\text{reject } H_0 \mid \mu_1 - \mu_2 = \delta]$

General picture:



Power in pictures

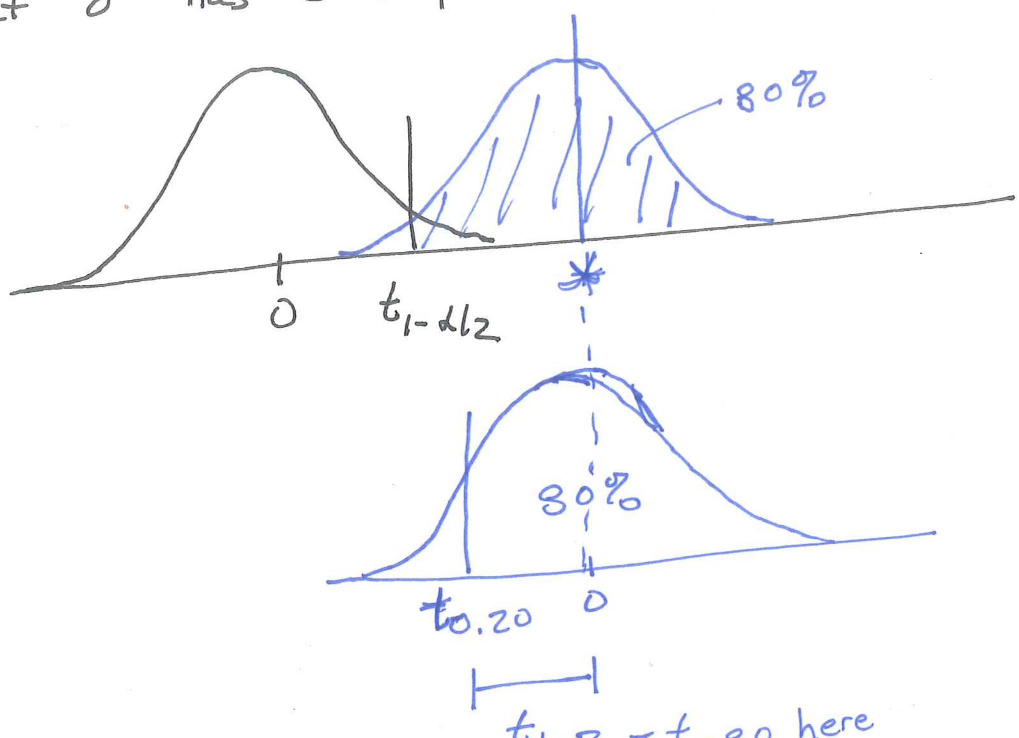
power depends on $\mu_1 - \mu_2$, call this δ

small δ (close to 0) $\Rightarrow$ low power (5–15%)

larger δ $\Rightarrow$ power 70–90%, perhaps

very large δ $\Rightarrow$ power > 99%

what δ has 80% power?



so * located at $t_{1-\alpha/2} + t_{1-\beta}$

Convert to a difference by multiplying by se $\Rightarrow$

$$\delta = \left(t_{1-\alpha/2} + t_{1-\beta}\right) se$$

Q.E.D.