

What if data have unequal sample sizes?

Reminder: What happens to a typical 2 way factorial when some observations are lost?

Not much so long as no missing cells

Tests are not orthogonal, require type III tests

All tests and estimates use the MSE

All tests and standard errors have integer df

No problems estimating marginal means or cell means or their se's

Imagine a split plot study with some lost observations

Corn water use efficiency study, 4 missing observations (out of 36)

one each from 0/16/1, 0/22/1, 0/28/1, 245/16/1 (p / water / n)

Two variance components: $\hat{\sigma}_{error}^2 = 4.77$, $\hat{\sigma}_{plot}^2 = 1.74$

Major consequence # 1: tests lose all the "magic" error terms

All the degrees of freedom are constructed and so have fractional df

Illustration: Main effect of water

Source	df	E MS	Error term	Error df		
				JMP	SAS	R
Water	2	$\sigma_e^2 + 2.259\sigma_p^2 + Q(water)$	$0.976 MS_{plot} + 0.024 MS_{error}$	4.33	4.33	4.75
Plot	5	$\sigma_e^2 + 2.314\sigma_p^2$				
Error	8	σ_e^2				

Estimated variance components are identical in JMP, R and SAS

Degrees of freedom are not all the same

Difference is the version of KR used to calculate df

When using REML / KR, split plot tests also get an adjustment - not 8 df

Doesn't usually make a practical difference to the p-value:

Water: JMP: 0.0022, SAS: 0.0022, R: 0.0015

P: JMP: 0.8667, SAS: 0.8667, R: 0.8659

Major consequence # 2: Each mean may have different df

df depends on the number and pattern of missing observations

Illustration: df for water marginal means

Water	Error df		
	JMP	SAS	R
16	4.68	4.68	5.04
22	4.12	4.12	4.46
28	4.12	4.12	4.46

Same estimates from JMP, SAS, and R. Slightly different se's.

Take home messages:

- Expect unusual results
- Fractional df for tests and estimates from unbalanced split plots
- Different results from software that uses different KR/Satterthwaite methods